REPORT ON "FREGEAN FRAGMENTS OF INTUITIONISTIC PROPOSITIONAL CALCULUS// WITH MIXED CONGRUENCE TYPE"

BY E. LYPKA

This dissertation deals with fragments of IPC in their algebraic form. More specifically it deals with equivalential subreducts of Heyting algebras with some other operation added. The relevance of these subreducts is explained adequately in the introduction and the question the candidate is trying to answer is the following: given a class of subreducts of Heyting algebras that is also congruence permutable and Fregean, when such a class present quotients of type 2 and 3 (in the Hobby-McKenzie sense).

This is a very hard problem, probably unsolvable at the current state of mathematical research, so some reduction is necessary. In this case the candidate chooses to focus on subreducts in a reduced language, involving only $\rightarrow, \wedge$ and 0, and in which the defining term operations have at most two variables. Under these hypotheses the candidate gives what I think it is a satisfactory answer and, given the obvious limitations of this kind of approach, it is probably the best answer that one can get at the moment.

The first chapter is devoted to the mandatory introductory material; it is exhaustive and well written. In the second chapter there is a description of two specific class of subreducts, EARS and EADS, that are congruence permutable, Fregean and of mixed type. This part expands the thesis work of S. Przybyło and half of it has already been published in a joint work with the candidate's advisor. These two classes (that turn out to be varieties) are particularly important, since they give exactly the *mixed-type flavor* that is sought after.

The search for the other subreducts starts in third chapter; the techniques are a clever mixture of standard mathematical reasoning, of combinatorial arguments and of some (very) brute force computations (sometimes computer assisted). It is not very easy to read but I fail to see how it could be made more readable; sometimes things are what they are and we have to live with it. The conclusion is that there are exactly 6 classes of subreducts with the desired properties (two of which are of course the aforementioned EARS and EADS), and five of them are varieties.

The final chapter of the dissertation contains a thorough description of the four missing classes. Here the techniques are more standard (with some heavy calculation involved) and the results are satisfactory.

The thesis is well written; I do have a couple of suggestions though. First in my opinion citing properly the thesis of S. Przbyło would be fair. Second I would add an Index (or Table of contents) which is missing; since it takes just one latex command to

do it I am sure it is an oversight. I did not check for misprints however just at page 2 there is an obvious one in the second sentence: the word "permutable" is missing in the second period of the sentence. In conclusion I believe that this dissertation fulfills all the reasonable requirements a dissertation should fulfill.

Finally let me give some (unsolicited) advice. The candidate mentions that he will continue to investigate this kind of structures but I urge caution. I understand that, after studying something for years, one is tempted to exploit it to the maximum. However cornering oneself in a topic that, albeit interesting, is rather *niche* is not something I would advise a young researcher to do...

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