CHENNAI MATHEMATICAL INSTITUTE
https://www.cmi.ac.in
(a University under Section 3 of the UGC Act, 1956)

Krishna Hanumanthu
Professor

June 8, 2026

Report on Ph.D thesis

Name of the candidate: Łukasz Merta
Affiliation: University of the National Education Commission, Krakow
Name of the thesis: Geometric properties of arrangements of low-degree curves

In this Ph.D thesis, Mr. Łukasz Merta studies arrangements of curves in the complex projective plane and gives explicit description of several interesting geometric objects.

The geometry of plane curves and their arrangements is an active area of algebraic geometry, lying at the intersection of singularity theory, combinatorics, and topology. A fundamental objective is to understand how the configuration of curves and the nature of their singularities determine algebraic and topological invariants of the complement and of the curves themselves. Inspired in part by the theory of free hyperplane arrangements, recent work has focused on extending notions such as freeness and related classification problems to arrangements of higher-degree curves, including conics and quartics. This line of research combines classical projective-geometric techniques with modern methods from commutative algebra and singularity theory.

The present thesis proves several nice new results in the area of plane curve arrangements. I briefly describe below the main results proved in this thesis.

The thesis contains three main chapters of original contributions.

In **Chapter 2**, the author considers Fermat curves F_n, $n \geq 3$. These are plane curves defined by the polynomial $x^n + y^n + z^n$. The author first describes the coordinates of *sextactic* points on F_n. These are points on F_n at which a conic meets it with intersection multiplicity at least 6. The coordinates of type 9 points on F_3 are also described. These are points where an irreducible cubic is tangent to F_3 with multiplicity 9. Further, the equations of the tangent conics and cubic at these points are described. Finally, certain arrangements of conic tangent to F_3 are studied.

The results in this chapter appeared in a paper in Journal of Algebra (written jointly with Maciej Zięba).

If C is a plane curve, a line L a *maximal tangency line* of C if C intersects L in a single point P. Such a point P is called *maximal tangency point* of C. **Chapter 3** looks at two special quartic curves, the Fermat quartic F_4 and the Komiya-Kuribayashi quartic, and describes their maximal tangency lines and points. Moreover, arrangements of conics tangent to these quartics are studied.

The results in this chapter appeared in a paper in Periodica Mathematica Hungarica (written jointly with Marcin Zieliński).

If C is a plane curve defined by $f \in S = \mathbb{C}[x, y, z]$ then the *module of logarithmic derivations* of C is the S-module

$$D(f) = \{\theta \in \mathrm{Der}_{\mathbb{C}}(S) \mid \theta(f) \in (f)\}.$$

The curve C is said to be *free* if $D(f)$ is a free S-module of rank 3. In **Chapter 4**, the author considers free arrangements of three smooth plane conics and classifies those arrangements which have ADE singularities. Free arrangements of curves satisfy certain combinatorial identities involving the local behaviour of their singularities (in terms of their Milnor and Tjurina numbers). Purely by this combinatorial analysis, the author reduces the possible free arrangements with ADE singularities to a small finite set. Then using geometric properties, it is shown that all but six of these cannot be realized by actual arrangements. At the end, the author gives explicit arrangements realizing the remaining six possibilities.

The results of this chapter appear in a preprint, written jointly with Filip Zieliński and Marcin Zieliński.

The thesis also includes an appendix containing the Singular code which was used for various computations in the thesis.

The results contained in this thesis are new, interesting and merit the award of a Ph.D degree from any good university. The thesis is also very well-written. Some minor suggestions are given in the next page for the author's consideration.

I am happy to recommend the acceptance of this thesis for the award of a Ph.D degree.

(Krishna Hanumanthu)

SIPCOT IT Park, Siruseri
Kelambakkam, Chennai
Tamilnadu 603 103, India

Tel. Off: +91 - 44 - 7196 1000
Fax : +91 - 44 - 2747 0225
E-mail: krishna@cmi.ac.in

Suggestions

Here are some mostly minor suggestions which the author may take into consideration and make suitable changes. I do not need to see a revised version of the thesis. I am happy to recommend the thesis in its present form.

- I could not find the definition of a *type 9* point in the thesis. Since it is an important object for the thesis (in chapter 2), it will be nice to explicitly add a definition in Preliminaries chapter (chapter 1).
- p13, last line: There is a typo in the description of inflection points. Perhaps the exponent should be $2k + 1$. Similarly, please check the equations of the tangent lines in line 4 of p14.
- It will be useful for a reader if references for some results are added (eg. Theorem 2.4 Abel's theorem, Fact 2.1, Fact 4.1)
- In Fact 4.1, shouldn't it be the equation of Q_3 that is being derived, instead of Q_2? I believe this Fact is being used to find the equation of Q_3 in later proofs.
- Proposition 4.8, 4.9, 4.10, non-existence of conic arrangements with the given weak combinatorics: in the beginning of the proofs, there is a figure giving the graph of a potential arrangement. The language suggests that we *assume* that this is the graph of a potential arrangement. Perhaps it is better to phrase it in such a way that any arrangement with given weak combinatorics has to have this graph (up to permutation?).
- Propositions 4.11, 4.12, 4.14, 4.16, 4.18, 4.19: I feel it is better to rephrase these statements. My suggestion is to first give the equations of Q_1, Q_2, Q_3 and say that the arrangement $\{Q_1, Q_2, Q_3\}$ has the given weak combinatorics. Then say something like *Further, any arrangements of conics with the given weak combinatorics is projectively equivalent to* $\{Q_1, Q_2, Q_3\}$. Then in the proof, first assert that $\{Q_1, Q_2, Q_3\}$ has the required weak combinatorics and the rest of the proof is exactly what is currently there.

Sincerely,

(Krishna Hanumanthu)

SIPCOT IT Park, Siruseri
Kelambakkam, Chennai
Tamilnadu 603 103, India

Tel. Off: +91 - 44 - 7196 1000
Fax : +91 - 44 - 2747 0225
E-mail: krishna@cmi.ac.in