

Abstract

This dissertation studies special configurations of points, lines, and curves in projective geometry through the algebra of their homogeneous ideals. The guiding theme is that classical incidence phenomena often have precise algebraic counterparts: symbolic powers detect assigned multiplicities, ordinary powers encode factorization constraints, Hilbert functions measure interpolation, and projections translate spatial configurations into plane complete intersections.

The first part of the dissertation gives the general mathematical background needed for the later chapters. It reviews finite subschemes of projective space, homogeneous ideals, Hilbert functions, complete intersections, symbolic powers, containment questions, Gröbner-basis membership tests, projections of finite sets, Weddle-type interpolation problems, and the group law on a smooth plane cubic. The discussion is written so that the later examples can be read as geometric constructions rather than as isolated computations.

The subsequent chapters treat four families of configurations. Yoshinaga's arrangement is used to study the failure of the containment $I^{(3)} \subseteq I^2$ for ideals of points in the plane. The $\mathbf{H}_4$ configuration gives an exceptional geproci set in $\mathbb{P}^3$, whose general projection to the plane is a complete intersection but which is not explained by the elementary grid construction. Weddle-type loci are discussed as vertex loci of cones through fixed base points, with careful attention to the distinction between interpolation counts and proved degree statements. Finally, flexes, sextactic points, and type-9 points on the Fermat cubic are described using the tangent-residual map and torsion in the elliptic-curve group law.

Keywords: symbolic powers, containment problem, Harbourne–Huneke containment, line arrangements, complete intersections, geproci sets, Weddle surfaces, Fermat cubic, sextactic points, torsion points.